

Some Khovanov-Rozansky Computations

See <http://www.math.toronto.edu/~drorbn/Misc/KhovanovRozansky/>

My programs are admittedly pathetic; they were written mostly to verify that I understand the definitions correctly. They are inelegant and inefficient and they hardly compute anything. Even the trefoil knot seems too large for these programs. (Though what's computed is enough to convince me that I do understand the definitions, and may already be of some very minor interest to others). Nothing was checked too carefully and all may be wrong.

Startup:

We first have to load some programs: the package `KnotTheory``, and the files `SubQuotient.m` and `KRH.m`. If you are wise and they are all in the same directory, just load all three. On my "development" machine, where each one is saved in a different place, the following sequence is necessary:

```
SetDirectory["c:/drorbn/projects/KhovanovRozansky/"];
AppendTo[$Path, "c:/drorbn/projects/KAtlas/"];
Get["KnotTheory`"];
Get["../SubQuotient/SubQuotient.m"];
Get["KRH.m"];
n = 2;

Loading KnotTheory`...
```

Here are some $n=2$ computations for the Hopf link, `Link[2, Alternating, 1]`:

```
K = PD[Link[2, Alternating, 1]];
KnotTheory::loading : Loading precomputed data in PD4Links`.
```

Degree -8, height 0 through 2:

```
Dim[KRH[K, #, -8]] & /@ {0, 1, 2}
{0, 0, 0}
```

Degree -6, height 0 through 2:

```
Dim[KRH[K, #, -6]] & /@ {0, 1, 2}
{0, 0, 1}
```

Degree -4, height 0 through 2:

```
Dim[KRH[K, #, -4]] & /@ {0, 1, 2}
{0, 0, 1}
```

Degree -2, height 0 through 2:

```
Dim[KRH[K, #, -2]] & /@ {0, 1, 2}
{1, 0, 0}
```

Degree 0, height 0 through 2:

```
Dim[KRH[K, #, 0]] & /@ {0, 1, 2}
{1, 0, 0}
```

Degree 2, height 0 through 2:

```
Dim[KRH[K, #, 2]] & /@ {0, 1, 2}
{0, 0, 0}
```

(These numbers agree with the ordinary Khovanov homology for the Hopf link, as expected for n=2)

Here are some n=2 computations for the trefoil knot, Knot[3,1]:

```
n = 2; K = PD[Knot[3, 1]];
KnotTheory::loading : Loading precomputed data in PD4Knots`.
Dim[KRH[K, #, -11]] & /@ {0, 1, 2, 3}
{0, 0, 0, 0}
Dim[KRH[K, #, -9]] & /@ {0, 1, 2, 3}
{0, 0, 0, 1}
Dim[KRH[K, #, -7]] & /@ {0, 1, 2, 3}
{0, 0, 0, 0}
Dim[KRH[K, #, -5]] & /@ {0, 1, 2, 3}
{0, 0, 1, 0}
```

(These numbers agree with the ordinary Khovanov homology for the trefoil knot, as expected for n=2)

Here are some $n=3$ computations for the Hopf link, `Link[2, Alternating, 1]`:

The easiest way to change n is to restart mathematica:

```
Exit[]

SetDirectory["c:/drorbn/projects/KhovanovRozansky/"];
AppendTo[$Path, "c:/drorbn/projects/KAtlas/"];
Get["KnotTheory`"];
Get["../SubQuotient/SubQuotient.m"];
Get["KRH.m"];
n = 3;

Loading KnotTheory`...

K = PD[Link[2, Alternating, 1]];

KnotTheory::loading : Loading precomputed data in PD4Links`.

Dim[KRH[K, #, -12]] & /@ {0, 1, 2}

{0, 0, 0}

Dim[KRH[K, #, -10]] & /@ {0, 1, 2}

{0, 0, 1}

Dim[KRH[K, #, -8]] & /@ {0, 1, 2}

{0, 0, 2}

Dim[KRH[K, #, -6]] & /@ {0, 1, 2}

{0, 0, 2}

Dim[KRH[K, #, -4]] & /@ {0, 1, 2}

{1, 0, 1}

Dim[KRH[K, #, -2]] & /@ {0, 1, 2}

{1, 0, 0}

Dim[KRH[K, #, 0]] & /@ {0, 1, 2}
```

Here are some $n=3$ computations for the trefoil knot, `Knot[3,1]`:

```
K = PD[Knot[3, 1]];

KnotTheory::loading : Loading precomputed data in PD4Knots`.
```

Degree -14, height 0 through 3:

```
Dim[KRH[K, #, -14, Parity → 0]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 1}
```

```
Dim[KRH[K, #, -14, Parity → 1]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 0}
```

Degree -12, height 0 through 3:

```
Dim[KRH[K, #, -12, Parity → 0]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 1}
```

```
Dim[KRH[K, #, -12, Parity → 1]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 0}
```

Degree -10, height 0 through 3 is already quite sad:

```
Dim[KRH[K, #, -10, Parity → 0]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 0}
```

```
Dim[KRH[K, #, -10, Parity → 1]] & /@ {0, 1, 2, 3}
```

```
$Aborted
```

Here are some n=4 computations for the trefoil knot, Knot[3,1]:

```
Exit[]
```

```
SetDirectory["c:/drorbn/projects/KhovanovRozansky/"];
```

```
AppendTo[$Path, "c:/drorbn/projects/KAtlas/"];
```

```
Get["KnotTheory`"];
```

```
Get["../SubQuotient/SubQuotient.m"];
```

```
Get["KRH.m"];
```

```
n = 4;
```

```
Loading KnotTheory`...
```

```
K = PD[Knot[3, 1]];
```

```
KnotTheory::loading : Loading precomputed data in PD4Knots`.
```

```
Dim[KRH[K, #, -21, Parity → 0]] & /@ {0, 1, 2, 3}
```

```
{0, 0, 0, 0}
```

```
Dim[KRH[K, #, -21, Parity → 1]] & /@ {0, 1, 2, 3}
{0, 0, 0, 0}

{Dim[KRH[K, #, -19, Parity → 0]] & /@ {0, 1, 2, 3},
 Dim[KRH[K, #, -19, Parity → 1]] & /@ {0, 1, 2, 3}}
{{0, 0, 0, 1}, {0, 0, 0, 0}}

{Dim[KRH[K, #, -17, Parity → 0]] & /@ {0, 1, 2, 3},
 Dim[KRH[K, #, -17, Parity → 1]] & /@ {0, 1, 2, 3}}
{{0, 0, 0, 1}, {0, 0, 0, 0}}

{Dim[KRH[K, #, -15, Parity → 0]] & /@ {0, 1, 2, 3},
 Dim[KRH[K, #, -15, Parity → 1]] & /@ {0, 1, 2, 3}}
```

No more memory available.

Mathematica kernel has shut down.

Try quitting other applications and then retry.